

$$\textcircled{1} \quad \begin{aligned} \lambda x + y &= \lambda^2 \\ x + \lambda y &= 1 \end{aligned}$$

$$\frac{a_1}{a_2} = \frac{\lambda}{1}, \quad \frac{b_1}{b_2} = \frac{1}{\lambda}, \quad \frac{c_1}{c_2} = \frac{\lambda^2}{1}$$

For no solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{\lambda}{1} = \frac{1}{\lambda} \neq \frac{\lambda^2}{1}$$

$$\frac{\lambda}{1} = \frac{1}{\lambda}$$

$$\Rightarrow \lambda^2 = 1$$

$$\Rightarrow \lambda = \pm 1$$

using $\lambda = 1$

$$\frac{a_1}{a_2} = \frac{1}{1}$$

$$\frac{b_1}{b_2} = \frac{1}{1}$$

$$\frac{c_1}{c_2} = \frac{1}{1}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

rejected

$$\lambda = -1$$

$$\frac{a_1}{a_2} = -1$$

$$\frac{b_1}{b_2} = -1$$

$$\frac{c_1}{c_2} = 1$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \boxed{\lambda = -1}$$

For in. Many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{\lambda}{1} = \frac{1}{\lambda} = \frac{\lambda^2}{1}$$

$$\frac{\lambda}{1} = \frac{1}{\lambda}$$

$$\Rightarrow \lambda^2 = 1$$

$$\Rightarrow \lambda = \pm 1$$

$$\frac{1}{\lambda} = \frac{\lambda^2}{1}$$

$$\Rightarrow \lambda^3 = 1$$

$$\Rightarrow \lambda = 1$$

$$\boxed{\therefore \lambda = 1}$$

For unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{\lambda}{1} \neq \frac{1}{\lambda}$$

$$\Rightarrow \lambda^2 \neq 1$$

$$\Rightarrow \lambda \neq \pm 1$$

any real value
except ± 1