

## CBSE - HOTS solutions by Dev Anoop (Bathinda)

If  $xyz = 1$ , then show that

$$(1 + x + y^{-1})^{-1} + (1 + y + z^{-1})^{-1} + (1 + z + x^{-1})^{-1} = 1$$

Answer

$$xyz = 1$$

$$\Rightarrow y = \frac{1}{xz} \text{ or } y^{-1} = xz$$

$$\begin{aligned} \text{LHS} &= (1 + x + y^{-1})^{-1} + (1 + y + z^{-1})^{-1} + (1 + z + x^{-1})^{-1} \\ &= (1 + x + xz)^{-1} + (1 + y + xy)^{-1} + (1 + z + yz)^{-1} \\ &= \frac{1}{1 + x + xz} + \frac{1}{1 + \frac{1}{xz} + \frac{1}{z}} + \frac{1}{1 + z + \frac{1}{x}} \\ &= \frac{1}{1 + x + xz} + \frac{1}{\frac{xz + 1 + x}{xz}} + \frac{1}{\frac{xz + 1 + x}{x}} \\ &= \frac{1}{1 + x + xz} + \frac{xz}{xz + 1 + x} + \frac{x}{xz + 1 + x} \\ &= \frac{1 + x + xz}{1 + x + xz} \\ &= 1 \end{aligned}$$